**Hypothesis Testing**

* Hypothesis testing
  + To conduct an experiment to determine how credible a statement is in light of observed data
* Types of errors
  + Type I error: reject H0 when it is true (when it shouldn’t be rejected)
    - E.g. convicting an innocent person of a crime
    - P(type I) = α
  + Type II error: do not reject H0 when it is false (when it should be rejected)
    - E.g. letting a guilty person go free
    - P(type II) = 1 − β
    - β = power of the test = (correctly) reject H0 when it is false
* Null hypothesis − the default hypothesis; a statement about the parameter/population of interest
  + Choose H0 to be the more likely outcome
  + E.g. H0: p = ½
* Alternative hypothesis − the hypothesis to support if H0 is rejected
  + E.g. H1/HA: p < ½ or p > ½ or p ≠ ½
  + Use one of the alternatives, depending on the problem given
* Ex: let population ~ X ~ N(μ, σ)
  + Sample = X1 … Xn
  + Step 1 – state H0 and HA
    - Test H0: μ = 3 vs. HA: μ ≠ 3 or μ > 3 or μ < 3
  + Step 2 – choose a test statistic
    - Test statistic/discrepancy measure – function of the data X1 … Xn that measures the degree of agreement between the data and the null hypothesis
    - i.e. T(X1 … Xn; θ)
    - T = 0 → strong agreement
    - T >> → poor agreement
  + Step 3 – find observed value of T
    - T\_obs = T(x1 … xn; θ)
  + Step 4 – critical value approach/P-value approach (see below)
  + Step 5 – compare T\_obs and t\_crit
    - Reject H0 if T\_obs falls into critical region
    - If right-sided i.e. HA: θ > θ0 → reject if observed > critical value
    - If left-sided i.e. HA: θ < θ0 → reject if observed < critical value
    - If two-sided i.e. HA: θ ≠ θ0 → reject if observed > critical or observed < −(critical)
* **Critical value approach**
  + T as a r. v. − possible values of T are divided into 2 regions:
    - Acceptance region – i.e. H0 is true
    - Rejection/critical region – i.e. H0 is false
  + If H0: θ = θ0 vs. HA: θ > θ0 → alternative is right-sided
    - i.e. critical region is right-sided → accept | reject
  + If HA: θ < θ0
    - i.e. critical region is left-sided → reject | accept
  + If HA: θ ≠ θ0
    - i.e. critical region is two-sided → reject | accept | reject
    - critical values are usually symmetric
  + t\_crit = critical values of the test = values at the boundaries between the acceptance & rejection regions (found from table)
* Ex: toss a coin 20 times
  + Test H0: p = 0.5 vs. Ha: p < 0.5 where p = P(head) (coin is symmetric)
  + Under H0, X(# of heads) ~ Bin(20, 0.5)
  + Suppose observed value = 8 (given)
  + Since Ha is left-sided, critical region is below some x\_crit
    - Area of critical region = α
  + Suppose α = 5% = 0.05 (given)
  + Need to find x\_crit:
    - Reject H0 if P(X ≤ x\_crit) ≤ α
    - → P(X ≤ x\_crit) ≤ 0.05
  + Find x\_crit from binomial distribution table
    - x-crit = 5 for n = 20, p = 0.5, and P = 0.05
  + Since x\_obs > t\_crit we do not reject H0
* Ex: same situation
  + Test H0: p = 0.5 vs. Ha: p > 0.5
  + Reject H0 if
    - P(X ≥ x\_crit) ≤ α
    - → P(X < x\_crit) ≥ 1 – α
    - → P(X < x\_crit) ≥ 0.95
  + x\_crit = 14 for n = 20, p = 0.5, and P = 0.95
* Hypothesis testing for the mean
  + Let population ~ X ~ N(μ, σ)
  + Step 1 − test H0: μ = μ0 vs. H1: μ > μ0 vs. H2: μ < μ0 vs. H3: μ ≠ μ0
  + Step 2 – test statistic:
    - T = Z = (X-bar − μ0)/(σ/√n) if σ is known
    - T = Z = (X-bar − μ0)/(S/√n) if σ is unknown and n ≥ 30
    - T = t = (X-bar − μ0)/(S/√n) ~ tn-1 if σ is unknown and n < 30
  + Step 3 – find Z\_obs or t\_obs
  + Step 4 – find critical value given α
    - H1 → right-sided
      * Z\_crit = Z\_α
    - H2 → left-sided
      * Z\_crit = Z\_α
    - H3 → two-sided
      * Total area of critical region = α → α/2 on either side
      * Z\_crit = Z\_α/2
  + Step 5 – compare observed and critical values